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# Flexible Design-Planning of Supply Chain Networks

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*Nowadays market competition is essentially associated to supply chain (SC) improvement. Therefore, the locus of value creation has shifted to the chain network. The strategic decision of determining the optimal SC network structure plays a vital role in the later optimization of SC operations. This work focuses on the design and retrofit of SCs. Traditional approaches available in literature addressing this problem usually utilize as departing point a rigid predefined network structure which may restrict the opportunities of adding business value. Instead, a novel flexible formulation approach which translates a recipe representation to the SC environment is proposed to solve the challenging design-planning problem of SC networks. The resulting mixed integer linear programming model is aimed to achieve the best NPV as key performance metric. The potential of the presented approach is highlighted through illustrative examples of increasing complexity, where results of traditional rigid approaches and those offered by the flexible framework are compared. The implications of exploiting this potential flexibility to improve the SC performance are highlighted and are the subject of our further research work.* © 2009 American Institute of Chemical Engineers *AIChE J,* 55: 1736–1753, 2009
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## Introduction

A SC may be defined as a collaborative network wherein a number of various business entities work together in an effort to: (1) acquire raw materials, (2) convert these raw materials into specified final products, and (3) deliver these final products to retailers. This chain is traditionally characterized by a forward flow of materials and a backward flow of information.[1] Usually in SC literature, business entities are classified by echelon type (e.g., suppliers, manufacturers, distributors, retailers).

Subsequently, supply chain management (SCM) can be defined as the management of material, information, and financial flows through a SC that aims at producing and delivering products or services to consumers.[2] The main objectives usually are to achieve desired consumer satisfaction levels and/or satisfactory financial returns by synchronizing and coordinating the SC members activities. The need for such coordination grows out of several trends in the marketplace, such as the globalization of market economies. Business global perspective has led to the availability of a vast set of alternative sources of materials and other inputs as well as a wider array of potential customers. Customers' changing expectations regarding value of goods and services, combined with advances in technology and the availability of information, have driven the formation of interorganizational networks.[3]

The SC modeling problem is very complex. In practice, it is usually helpful to use the time dimension to establish a hierarchical order so as to facilitate SC coordination. At the top level, long-term or strategic planning affects the achievement of goals over sourcing and investment decisions. Long-term plans generally concern the enterprise infrastructure and links with external organizations in an aggregated manner. Once implemented, plans cannot be easily altered at this level because their implementation is typically capital

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intensive and time consuming. In terms of organizational structure, they are usually under the control of shareholders and/or upper level management personnel. On the other hand, tactical or middle term planning results in decisions related to the magnitude of material flows across the manufacturing/distribution network that is established by the strategic level. Finally, short term or operational planning is characterized by those many decisions that must be made for daily or weekly activities.

The need to support decision making and improve operations over all levels has lead to the development of several SC models. This article is focused on modeling the strategic-tactical decision level of SCM which has been an active research topic in the last 2 decades. Brown et al.[4] present an early example of a production/distribution network design study in the process industries where the biscuit division of Nabisco is considered. Their model involves the establishing or shutting-down of plants, the assignment of locations to plants, and the products assignment to facilities. Cakravastia et al.[5] present a two-level mathematical model for supplier selection in designing a SC network. The objective considered is to minimize the level of customer dissatisfaction, which is evaluated by two performance criteria: price and delivery lead time. Kallrath[6] describes a tool for simultaneous strategic and operational planning in a multisite production network. The author aims at optimizing the total net profit of a global network, where key decisions include: operating modes of equipment in each time period, production and supply of products, minor changes to the infrastructure (e.g., addition and removal of equipment from sites), and raw material purchases and contracts. Bok et al.[7] propose a multiperiod SC optimization model for operational planning of continuous flexible process networks, where sales, intermittent deliveries, production shortfalls, delivery delays, inventory profiles, and job changeovers are taken into account. Intermediate product flows among manufacturing sites are considered in the process networks, however process allocation to sites and SC components links are fixed because a planning problem is being addressed. Later on, the flexible process networks framework is extended to tackle SC design problems by You and Grossmann.[8] They address SC optimization under demand uncertainty considering responsiveness and net present value as the objectives to be maximized. The authors quantitatively analyze how network configurations may have an effect on the responsiveness to market changes. By using a probabilistic model for stock-out, the expected lead time is proposed as a quantitative measure of SC responsiveness. The production process network (from raw material to final products) may be decoupled in various processing "sub-trains." These subtrains can take place at different locations; however, a potential process SC network superstructure must be given a priori. Ferrio and Wassick[9] present an approach which is aimed to the redesign of existing SC networks. Their model consists in a single period network design MILP model for multiproduct SCs considering three echelons (processing plants, distribution centers, and customers). Direct shipping and product flows among plants and/or distribution centers are taken into account, but the potential linkages must be predefined. Their approach does not take into account the process network required to produce the final products.

The complexity of designing SC networks is increased by the dynamic market environment where their activities are performed (e.g., volatile demand, prices). Therefore, consideration of uncertainty has induced important research in SC strategic decision making. Tsiakis et al.[10] propose a model based on a detailed mathematical programming formulation that addresses the SC design under demand uncertainty. Their objective is the minimization of the total annualized cost taking into account both infrastructure and operating costs. A stochastic programming approach based on a recourse model with two stages is proposed also by Guillén et al.[11] to incorporate demand uncertainty within the design procedure. Their approach enables to consider and manage the financial risk associated to the different design options, resulting in a set of Pareto optimal solutions that can be used for decision-making. Mele et al.[12] consider a SC agent-oriented simulation system to solve the retrofit and design problem of a production/distribution network under uncertainty. The starting point is a set of possible network structure options. The performance of each SC configuration is assessed through a multiagent model that is coupled with a genetic algorithm in order to optimize the operation variables associated to each design candidate.

Otherwise, recent advances in Process Systems Engineering (PSE) have focused on devising modeling strategies that integrate decisions of distinct business functions into a global model. SC design has not been apart from this trend. Oh and Karimi[13] introduce and classify the major regulatory factors that may influence strategic decisions in the design and operation of chemical SCs. The effects of two important regulatory factors, corporate tax and import duty, in the capacity-planning decisions are reported. Their model treats the sizes of capacity expansions and new facility capacities as decision variables and incorporates SC operation decisions, such as sourcing of raw materials and facility production rates, which can eventually affect the strategic capacity-planning decisions. Hugo and Pistikopoulos[14] address the environmentally conscious process selection problem for the long-term planning and design of chemical SC networks. A mathematical programming-based methodology is presented for the explicit inclusion of Life Cycle Assessment as one of the criteria to be considered in the strategic investment decisions related to the design and planning of SC networks. Very recently, Laínez et al.[15] address the SC design problem considering financial issues. Their model embeds a capital budgeting formulation enabling the quantitative assessment of the firm's value. The corporate value of the firm (CV) is proposed as objective function, which is calculated using the discounted-free-cash-flow method.

Reviews of works related to strategic SC models can be found in Beamon[1], Vidal and Goetschalckx,[16] Schmidt and Wilhelm,[17] Meixell and Gargeya,[18] and Shah.[19]

Evidently, a SC network is comprised by lateral links, reverse loops, two way exchanges, and so forth, encompassing the upstream and downstream activity.[20] Notwithstanding, common characteristics of most existing SC approaches are that among SC components exclusively vertical flows that have only one direction are considered (see Figure 1) and a predefined superstructure is necessary. Vertical flows are understood as those flows between SC components that belong to consecutive echelons type (e.g., flows that go from

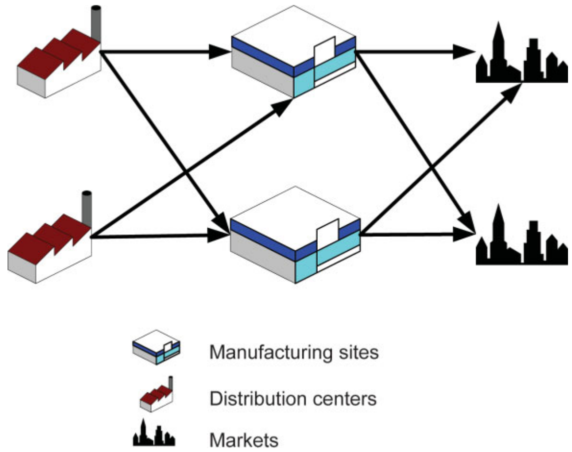



**Figure 1. Traditional framework for SC design.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

wholesalers to retailers, from production plants to wholesalers). Instead, this work proposes a flexible SC design-planning formulation (see Figure 2) whose distinctive features are that (i) considers all feasible links and material (raw material, intermediate, and final products) flows among the potential SC components inherently and (ii) does not need any pre-established process network superstructure so that the sub-trains (if any) in which production process is decoupled and their location are determined by the model. Regarding the latter feature, our model does merely require as input the SC production process recipe representation. The abovementioned features provide opportunities to select a value-efficient allocation of production-distribution activities which may lead to more profitable SC designs. Furthermore, a more appropriate description of manufacturing processes at the SC level is achieved by translating a recipe representation to the SC environment.

Finally, it is important to mention that flexibility has been recognized as a key strategy for efficiently improving responsiveness of production systems facing demand uncertainty. Jordan and Graves[21] analyze the process flexibility which can be understood as the ability to build different types of products in the same capacitated processing resource at the same time. They propose guidelines to identify the best way to add process flexibility to a network of plants and conclude that it is creating longer product-plant ‘‘chains’’ what results in flexibility benefits, instead of building diverse products in a single plant. The authors define a ‘‘chain’’ as a group of products and plants which are all connected, directly or indirectly, by product assignment decisions. All products within a chain share that chain's capacity, even without each plant building all products. Later on, Graves and Tomlin[22] extend this work to understand the role of process flexibility in general multistage SCs. Their analysis disregards economic issues, then comparisons among different configurations are carried out using capacity utilization and demand satisfaction as performance indicators. Our approach is similar in the sense that includes the analysis of which products should be processed at each plant but also considering economic issues. Besides our approach is deterministic; hence, demand uncertainty is not taken into account. Certainly, the presented approach can be further extended to a stochastic model so that more responsive SC designs can be obtained; although that extension is out of the present work scope and objectives. This work focuses in how improved SC designs are obtained by more properly considering: the connectivity among SC components and the selection of processes (stages) to be installed at each location.

The article is organized as follows. Problem Statement section presents a formal definition of the SC design-planning problem. In Mathematical Formulation section, the corresponding mathematical formulation is presented and explained while its performance is illustrated in Examples section. Finally, some conclusions about this work are drawn in Final Considerations section.

## Problem Statement

The scope of the SC network design problem is typically to determine the optimal manufacturing and distribution network for the entire product line of a company according to a pre-established economic objective. The most common approach is to formulate a large-scale mixed-integer linear program that captures the relevant fixed and variable operating costs for each facility and each major product family. The fixed costs are usually associated with the investment and/or overhead costs for opening and operating a facility, or with placing a product family in a facility. The variable costs include not only the manufacturing, procurement, and distributions costs but also the tariffs and taxes that depend on the network design. The network design problem usually focuses on the design of two or three major echelons in the SC in order to handle tractable problems. Because of the nature of the problem being solved, optimum network design is typically reanalyzed every 2 to 5 years.[23]

As it has been already mentioned in the introduction, the majority of works in the PSE field related to chemical SC network design relies on the concept of fixed ‘‘echelons’’

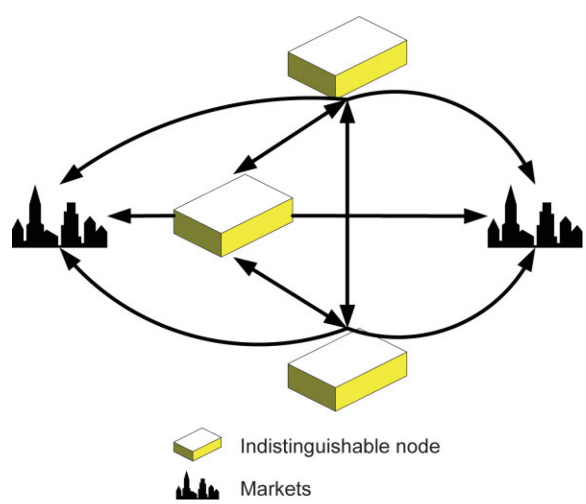



**Figure 2. Flexible framework for SC design.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

(Figure 1); they assume a given fundamental structure for the network in terms of the echelons involved (i.e., consecutive SC components connected by vertical flows). Thus, a rather rigid structure is imposed on the SC network and the design procedure focuses on the determination of the number of components in each echelon and the connectivity between components in adjacent echelons. However, changes in the fundamental structure of the network may sometimes lead to economic benefits that far exceed what can be achieved merely by changing the number of components and/or the connectivity within an existing structure.[19]

The multiperiod deterministic model that is developed next provides a flexible framework for the design of SCs. The model assumes that equipment is available for eventual installation at potential locations and assists in their selection. Furthermore, the model allows for the expansion of plant equipment capacities, not only in the first planning period but also during any other period in which managers believe that opportunities for investing on facilities may result in a more favorable performance. The problem can be stated as follows (the following data is assumed to be known in advance):

Input data

- a fixed time horizon;
- a set of products;
- a set of markets in which products are available to customers and their nominal demand;
- a set of potential geographical sites for locating facilities;
- a set of potential equipment for manufacturing the different products;
- lower and upper bounds for the increment of equipment and storage capacity;
- product recipes (mass balance coefficients and utilization of production resources);
- suppliers capacity;
- minimum utilization rate of installed capacity;
- direct cost parameters such as production, handling, transportation and raw material costs;
- price for every product in each market during the time horizon;
- relationship between capital investment and facility capacity;
- relationship between indirect expenses and facility capacity;

The goal is to determine:

- the facilities to be opened;
- the increase in facility capacity in each time period;
- the linkages among facilities;
- the assignment of manufacturing and distribution tasks to the network nodes;
- the amount of final products to be sold;

such that an economic performance metric to be evaluated at the end of the planning horizon is maximized.

The model utilizes a uniform discrete time formulation. It is assumed that demand is satisfied (i.e., sales execution) at the end of each time period in which the planning horizon is divided. It is also noteworthy to mention that it is assumed that some of the demand can be left unsatisfied because of limited production capacity.

## Mathematical Formulation

### *Design and planning model*

The State-Task-Network (STN) representation[24] has been utilized to formulate the problem of production scheduling in multipurpose plants as a MILP problem. An important feature of this representation is that both the individual operations (tasks) and the feedstocks, intermediate and final products (states) are included explicitly as network nodes. Processes involving sharing of raw materials and intermediates, batch splitting and mixing and recycles of material, can be represented unambiguously in such networks (see Figure 3). In this work, the STN concept is expanded to formulate the flexible design-planning SC network model. The proposed model implements the flexible echelons concept. Hence, the connectivity between echelons is not imposed and a facility may play the role of either a processing and/or a distribution site. Material flows among facilities are allowed even if they belong to the same echelon type. Moreover, raw, intermediate, and final material flows between facilities appear. The STN formulation has been enhanced to handle multiple site locations so that the abovementioned aspects can be addressed. These aspects rely on the definition of variable $P_{ijff't}$ which is analog to the batch size in Kondili's original scheduling model ($P_{ijff't}$ represents the amount of task $i$ performed in equipment $j$ which receives input materials from site $f$ and "delivers" output materials to site $f'$ during period $t$). Furthermore, production ($i \notin Tr$) and distribution ($i \in Tr$) tasks are strictly separated. Figure 4a depicts how the production rates are managed by the model. Certainly, a production task receives and delivers material within the same site. In case of distribution tasks, facilities $f$ and $f'$ must be different (Figure 4b). As the reader can observe, a key characteristic of the model is that each task is associated to an origin and a final site location ($f$, $f'$). Thereby, a distribution center $f$ can be easily identified because all its associated production rates are equal to zero ($P_{ijfft} = 0$).

The design-planning model is explained in detail next. The equations have been categorized into four groups; namely (i) mass balances, (ii) design, (iii) capacity, and (iv) markets and suppliers constraints.

*Mass Balances.* Mass balances must be satisfied at each of the nodes that integrate the SC network. Equation 1

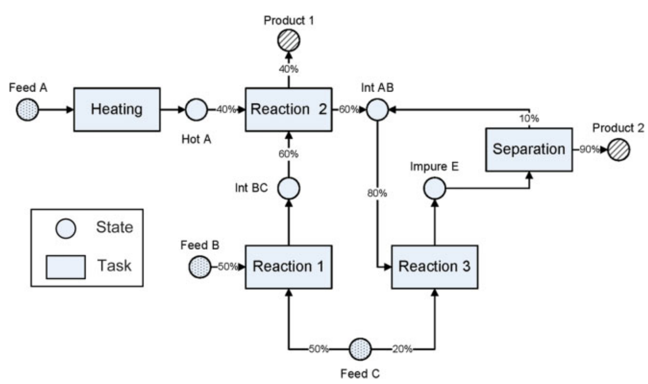



**Figure 3. STN example.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

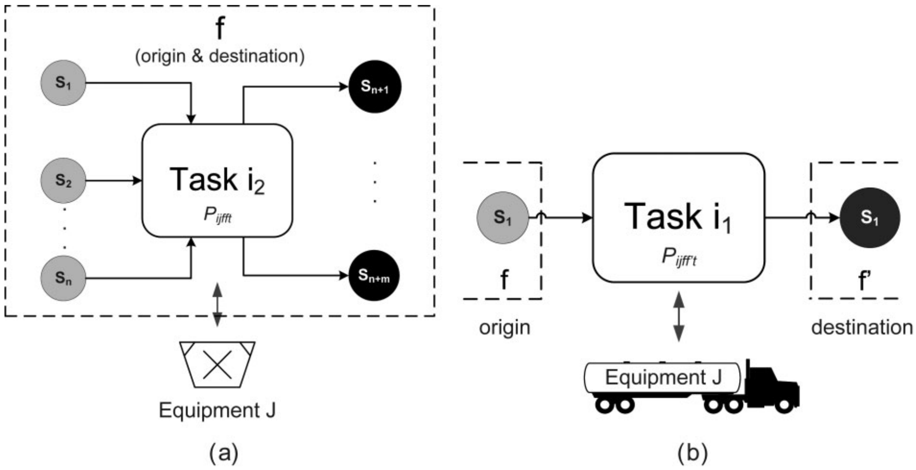



**Figure 4. SC activities State-Task representation (a) Production task (b) Distribution task.**

denotes the material balance for each state $s$ in every facility $f$ at period $t$. Inventory change ($S_{sft} - S_{sft-1}$) of consecutive periods ($t-1$,$t$) must be equal to the difference between the amount of material $s$ ($\alpha_{sij}$ $P_{ijf'ft}$) produced/transported by tasks ($i \in T_s$), whose destination is facility $f$, and the amount ($\overline{\alpha}_{sij} P_{ijff't}$) consumed by tasks ($i \in \overline{T}_s$), whose origin is facility $f$. Here, $\alpha_{sij}$ and $\overline{\alpha}_{sij}$ represent the mass fraction coefficients of material $s$ for task $i$ performed in equipment $j$. It is noted that this equation should be merely applied to those equipment technologies which are suitable to perform task $i$ ($J_i$) and only if these technologies can be installed on the corresponding facility ($\hat{J}_f$). $M$ and Sup are the subsets of locations where markets and suppliers are placed, respectively.

$$S_{sft} - S_{sft-1} = \sum_{f'} \sum_{i \in T_s} \sum_{j \in (J_i \cap \hat{J}_{f'})} \alpha_{sij} P_{ijf'ft} - \sum_{f'} \sum_{i \in \overline{T}_s} \sum_{j \in (J_i \cap \hat{J}_f)} \overline{\alpha}_{sij} P_{ijff't} \quad \forall s, f \notin (\mathrm{Sup} \cup M), t \tag{1}$$

In the process industry, there might be cases in which some intermediate material $s$ cannot be transfered between facilities due to its instability. Distribution tasks associated to these materials can be restricted within the model by using the following constraint. Here, $S^I$ represents the unstable materials set.

$$P_{ijff't} = 0 \quad \forall s \in S^I, i \in (Tr \cup T_s), f' \neq f, t \tag{2}$$

*Design Constraints.* Capacity and facilities location constraints are stated next. Similar formulations can be found in Hugo and Pistikopoulos[14] and Laínez et al.[15] Here, $\mathrm{FJ}_{jft}$ and $\mathrm{FS}_{ft}$ represent the equipment $j$ total capacity in site $f$ and the total storage capacity in site $f$ accounting for decisions made until period $t$, respectively. Moreover, variables $\mathrm{FJE}_{jft}$ and $\mathrm{FSE}_{ft}$ denote the facilities capacity expansion decision at period $t$. In these variables, it is important to notice that period $t$ reflects the period when the expansion decision is taken. The model is general enough to address not only the design-planning of a new SC but also the retrofitting of an existing SC network. In the latter case, the problem should be formulated by fixing at the initial period ($t = 0$) the value of the variables representing the facilities capacity according to the initial network topology. No installation/construction time is considered for the initial network.

Equations 3 and 4 are added to control the changes in the facilities capacity over time. These constraints include binary variables $V_{jft}$ and $X_{ft}$, which take a value of 1 if the facility being represented (either the equipment $j$ at site $f$ or the storage) is expanded in capacity, otherwise is set to zero. The capacity increments are bounded in the range [$\mathrm{FJE}^{\mathrm{L}}_{jft}$, $\mathrm{FJE}^{\mathrm{U}}_{jft}$] and [$\mathrm{FSE}^{\mathrm{L}}_{ft}$ ,$\mathrm{FSE}^{\mathrm{U}}_{ft}$], which represent the realistic intervals where they must fall.

$$V_{jft}\mathrm{FJE}^{\mathrm{L}}_{jft} \leq \mathrm{FJE}_{jft} \leq V_{jft}\mathrm{FJE}^{\mathrm{U}}_{jft} \quad \forall f \notin (\mathrm{Sup} \cup M), j \in \hat{J}_f, t \tag{3}$$

$$X_{ft}\mathrm{FSE}^{\mathrm{L}}_{ft} \leq \mathrm{FSE}_{ft} \leq X_{ft}\mathrm{FSE}^{\mathrm{U}}_{ft} \quad \forall f \notin (\mathrm{Sup} \cup M), t \tag{4}$$

Equations 5 and 6 are added to update the total capacity ($\mathrm{FJ}_{jft}$ and $\mathrm{FS}_{ft}$) by the amount increased during planning period $t$ ($FJE_{jft}$ and $FSE_{ft}$).

$$\mathrm{FJ}_{jft} = \mathrm{FJ}_{jft-1} + \mathrm{FJE}_{jft} \quad \forall f \notin (\mathrm{Sup} \cup M), j \in \hat{J}_f, t \tag{5}$$

$$\mathrm{FS}_{ft} = \mathrm{FS}_{ft-1} + \mathrm{FSE}_{ft} \quad \forall f \notin (\mathrm{Sup} \cup M), t \tag{6}$$

*Capacity Constraints.* Equation 7 forces the total production/distribution rate in each facility to be greater than a minimum desired capacity utilization ($\beta_{jf}$ $\mathrm{FJ}_{jft-\pi_{jf}}$) and lower than the available capacity ($\mathrm{FJ}_{jft-\pi_{jf}}$). In this equation, $\theta_{ijff'}$ represents the capacity utilization factor of equipment $j$ by task $i$. To go on, $\beta_{jf}$ expresses the minimum percentage of utilization of equipment $j$ at site $f$. Finally, $\pi_{jf}$ reflects the necessary time to install and set up equipment $j$ in facility $f$. It is noteworthy that the model considers that task $i$ is to be performed in equipment that is installed on the facility of "origin." Notice that those expansion decisions made in period $t - \pi$ are available to be utilized in period $t$.

$$\beta_{jf}\mathrm{FJ}_{jft-\pi_{jf}} \leq \sum_{f'} \sum_{i \in I_j} \theta_{ijff'} P_{ijff't} \leq \mathrm{FJ}_{jft-\pi_{jf}} \quad \forall f \notin (\mathrm{Sup} \cup M), j \in \hat{J}_f, t \tag{7}$$

In the same way, total inventory in facility $f$ is constrained to be equal to or lower than the available capacity ($FS_{ft-\hat{\pi}_f}$) in each period $t$ by Eq. 8. In this equation, $\hat{\pi}_f$ is the time required to install storage and material handling equipment, while $\upsilon_s$ holds for specific volume of material $s$.

$$\sum_{s} \upsilon_s S_{sft} \leq FS_{ft-\pi_f} \quad \forall f \notin (\mathrm{Sup} \cup M), t \tag{8}$$

*Markets and Suppliers.* Regarding markets and suppliers, the model assumes that markets and suppliers are placed at "different" locations from facilities. Hence, for modeling purposes, dummy locations are needed in case facilities can be installed on the same site a market and/or supplier is. In this way, it is followed the distribution tasks representation ($P_{ijff't}$) which is based on distinguishing the origin and destination locations. It is emphasized that despite the previous assumption the model permits different suppliers to be located at the same site. By Eq. 9, sales of final product $s \in$ FP carried out from facility location $f'$ to market $f \in M$ are estimated. Equation 10 states that sales in markets during period $t$ must be less than or equal to the demand. Also, a minimum customer service level (CSL) target for each product, which must be attained in all periods, is imposed by Eq. 11.

$$\mathrm{Sales}_{sf'ft} = \sum_{i \in (T_s \cap T_r)} \sum_{j \in (J_i \cap \hat{J}_f)} P_{ijf'ft} \quad \forall s \in \mathrm{FP}, f \in M,\ f' \notin M, t \tag{9}$$

$$\sum_{f' \notin M} \mathrm{Sales}_{sf'ft} \leq \mathrm{Dem}_{sft} \quad \forall s \in \mathrm{FP}, f \in M, t \tag{10}$$

$$\frac{\sum_{f \in M} \sum_{f' \notin M} \mathrm{Sales}_{sf'ft}}{\sum_{f \in M} \mathrm{Dem}_{sft}} \geq \mathrm{MinCSL}_s \quad \forall s \in \mathrm{FP}, t \tag{11}$$

Finally, the model also assumes a maximum availability of raw materials. Therefore, Eq. 12 forces the amount of raw material, $s \in$ RM, purchased from location $f \in$ Sup at each period $t$ to be lower than an upper bound, $A_{sft}$, given by physical limitations. In this expression, $R_f$ denotes the set of raw materials that can be provided from location $f$.

$$\sum_{f' \notin \mathrm{Sup}} \sum_{i \in (\bar{T}_s \cap Tr)} \sum_{j \in J_i} \bar{\alpha}_{sij} P_{ijff't} \leq A_{sft} \quad \forall f \in \mathrm{Sup},\ s \in R_f, t \tag{12}$$

### *Economic performance metrics formulation*

Many economic performance metrics have been utilized for determining the goodness of a SC network design. The most traditional metrics are profit, net present value (NPV), and total cost. The current business environment has led managers to become aware of the financial dimension of decision making. Thus, business managers are becoming more driven by the goal of enhancing shareholder value. By recognizing this fact, corporate value has been proposed by Laínez et al.[15] as a suitable financial indicator that is able to properly assess the trade-off between net operating income (i.e., profit) and capital efficiency (i.e., fixed assets and net working capital). In this section, expressions that provide the essential input to compute the aforementioned economic indicators are presented. Indeed, these expressions are the ones that allow the integration between the proposed design-planning model and a financial formulation. In this way, the model is general enough to suit different approaches. Slight modifications and/or extra equations will be required depending on the indicator selected as objective function. For the sake of clarity and to evidence the proposed model advantages, NPV is selected as performance indicator in the mathematical formulation of this work. NPV is one of the most widely used economic objectives when assessing strategic problems.

In the next subsections, expressions have been classified in three groups: (i) operating revenue, (ii) operating cost, and (iii) capital investment.

*Operating Revenue* Revenue is calculated by means of net sales which are the income source related to the normal SC activities. Thus, the total revenue incurred in any period $t$ can be easily computed from the sales of products executed in period $t$ as it is stated in Eq. 13.

$$\mathrm{ESales}_t = \sum_{s \in \mathrm{FP}} \sum_{f \in M} \sum_{f'} \mathrm{Sales}_{sf'ft} \mathrm{Price}_{sft} \quad \forall t \tag{13}$$

*Operating Cost*

*Indirect cost.* The total fixed cost of operating a given SC structure can be computed using Eq. 14. $\mathrm{FCFJ}_{jft}$ and $\mathrm{FCFS}_{ft}$ are the fixed unitary capacity cost for production equipment and storage, respectively. Regarding the installation times, a reasoning similar to the one in Capacity Constraints section is as follows:

$$\mathrm{FCost}_t = \sum_{f} \sum_{j \in \hat{J}_f} \mathrm{FCFJ}_{jft} \mathrm{FJ}_{jft-\pi_{jf}} + \sum_{f} \mathrm{FCFS}_{ft} \mathrm{FS}_{ft-\pi_f} \quad \forall t \tag{14}$$

*Direct cost.* The cost of purchases from supplier $e$, which is computed through Eq. 15, includes purchases of raw materials, transportation, and production resources. Let us notice that $e$ refers to supplier entity and not to supplier location $f$ ($f \in$ Sup).

$$\mathrm{EPurch}_{et} = \mathrm{Purch}^{\mathrm{rm}}_{et} + \mathrm{Purch}^{\mathrm{tr}}_{et} + \mathrm{Purch}^{\mathrm{prod}}_{et} \quad \forall e, t \tag{15}$$

The purchases ($\mathrm{EPurch}_{et}$) associated to raw materials made to supplier $e$ can be computed through Eq. 16. $\psi_{est}$ is the cost associated to raw material $s$ purchased from supplier $e$. Because it is assumed that several suppliers of raw materials may exists, Eq. 17 expresses that the total quantity of raw materials purchased in period $t$ must be equal to the sum of the amounts purchased from each supplier $e$.

$$\mathrm{Purch}^{\mathrm{rm}}_{et} = \sum_{s \in \mathrm{RM}_e} \sum_{f \in \mathrm{Sup}} \mathrm{Purch}_{esft} \psi_{est} \quad \forall e, t \tag{16}$$

$$\sum_{f'} \sum_{i \in (\bar{T}_s \cap Tr)} \sum_{j \in J_i} \bar{\alpha}_{sij} P_{ijff't} = \sum_{E_s} \mathrm{Purch}_{esft} \quad \forall s \in \mathrm{RM}, f \in \mathrm{Sup},\ t \tag{17}$$

Otherwise, for the sake of simplicity external transportation services as well as production resources are assumed to be "acquired" each of them from one unique supplier (i.e., $|E_{tr}| = |\hat{E}_{prod}| = 1$). This assumption can be easily relaxed to address more general cases. Production and transportation costs are determined by Eqs. 18 and 19, respectively. Here, $\rho^{tr}_{eff'}$ denotes the unitary transportation cost associated with sending products from location $f$ to location $f'$. $\tau^{ut1}_{iife}$ represents the unitary production cost associated to perform task $i$ in processing equipment $j$, whereas $\tau^{ut2}_{sfe}$ represents the unitary inventory costs.

$$\text{Purch}^{tr}_{et} = \sum_{i \in Tr} \sum_{j \in J_i} \sum_{f} \sum_{f'} P_{ijff't} \rho^{tr}_{eff'} \quad \forall e \in \tilde{E}_{tr}, t \tag{18}$$

$$\text{Purch}^{prod}_{et} = \sum_{f} \sum_{i \notin Tr} \sum_{j \in (J_i \cap \hat{J}_f)} P_{ijfft} \tau^{ut1}_{ijfe} + \sum_{s} \sum_{f \notin (Sup \cup M)} S_{sft} \tau^{ut2}_{sfe} \quad \forall e \in \hat{E}_{prod}, t \tag{19}$$

*Capital Investment*. Finally, the total investment on fixed assets is computed through Eq. 20. This equation includes the investment made to expand the equipment $j$ capacity in facility site $f$ at period $t$ ($\text{Price}^{FJ}_{jft}$ $\text{FJE}_{jft}$), plus the investment required to open a manufacturing plant in location $f$, in case it is opened at period $t$ ($I^{J}_{ft}$ $\text{JB}_{ft}$), plus the investment required to support distribution center capacity increase ($\text{Price}^{FS}_{ft}$ $\text{FSE}_{ft}$), plus the investment required to set a distribution center if it is opened at period $t$ ($I^{S}_{ft}$ $\text{JB}_{ft}$). Here, $\text{JB}_{ft}$ and $\text{SB}_{ft}$ are binary variables which take value of 1 in case the facility being represented, processing site or distribution center, starts construction in period $t$.

$$\text{FAsset}_t = \sum_{f} \left( \sum_{j} \text{Price}^{FJ}_{jft} \text{FJE}_{jft} + I^{J}_{ft} \text{JB}_{ft} \right) + \sum_{f} (\text{Price}^{FS}_{ft} \text{FSE}_{ft} + I^{S}_{ft} \text{SB}_{ft}) \quad \forall t \tag{20}$$

The following disjunctive expressions allow to define binary variables $\text{JB}_{ft}$ and $\text{SB}_{ft}$. The equivalent logic conditions state that if no equipment was decided to be installed in previous periods ($t' < t$) and some equipment is decided to be installed at the current period $t$, then decision to open facility $f$ is made at period $t$.

$$\left[ \neg \bigvee_{t' < t} \text{JB}_{ft'} \right] \wedge \left[ \bigvee_{j \in \hat{J}_f} V_{jft} \right] \Rightarrow \text{JB}_{ft} \quad \forall \notin (\text{Sup} \cup M), t$$

$$\left[ \neg \bigvee_{t' < t} \text{SB}_{ft'} \right] \wedge X_{ft} \Rightarrow \text{SB}_{ft} \quad \forall f \notin (\text{Sup} \cup M), t$$

Next, the previous disjunctions are transformed in mixed integer constraints. Equations 21 and 22 are to force definition of variable $\text{JB}_{ft}$, while Eqs. 23 and 24 restrict variable $\text{SB}_{ft}$.

$$\sum_{j \in J_f} \left( \sum_{t' \le t} \text{JB}_{ft'} - V_{jft} \right) \ge 0 \quad \forall f \notin (\text{Sup} \cup M), t \tag{21}$$

$$\sum_{t} \text{JB}_{ft} \le 1 \quad \forall f \notin (\text{Sup} \cup M) \tag{22}$$

$$\sum_{t' \le t} \text{SB}_{ft'} - X_{ft} \ge 0 \quad \forall f \notin (\text{Sup} \cup M), t \tag{23}$$

$$\sum_{t} \text{SB}_{ft} \le 1 \quad \forall f \notin (\text{Sup} \cup M) \tag{24}$$

Equation 25 represents the calculation of profit at period $t$. To conclude, NPV is computed by means of Eq. 26.

$$\text{Profit}_t = \text{ESales}_t - (\text{FCost}_t + \sum_{e} \text{EPurch}_{et}) \quad \forall t \tag{25}$$

$$\text{NPV} = \sum_{t} \left( \frac{\text{Profit}_t - \text{FAsset}_t}{(1 + \text{rate})^t} \right) \tag{26}$$

Thus, the SC network design-planning problem whose objective is to optimize NPV can be mathematically posed as follows:

$$\begin{aligned} &\underset{\mathscr{X}, \mathscr{Y}}{\text{Max}}\ \text{NPV} \\ &\text{subject to Eqs. } 1-26 \\ &\mathscr{X} \in \{0, 1\}; \mathscr{Y} \in \mathbb{R} \end{aligned}$$

here $\mathscr{X}$ denotes the model binary variables set, while $\mathscr{Y}$ represents the model continuous variable set.

## Examples

The capabilities of the flexible model are illustrated by solving four SC design-planning problems. The first three problems are illustrative examples which intend to demonstrate some of the special features of the flexible model. The last one constitutes a more sophisticated case study based on a real problem. The resulting MILP models have been solved to optimality in GAMS using CPLEX (11.0) on a computer with an 2.0 GHz Intel Core 2 Duo and 2GB RAM.

### *Illustrative examples*

*Example 1*. A SC design-planning problem comprising four potential locations for the processing sites and the distribution centers is presented. A planning horizon of five annual periods is considered. The STN representation of the production process is depicted in Figure 5. Two final products ($S4$ and $S5$) can be sold in two markets ($M1$ and $M2$). $S1$ and $S2$ are raw materials. A set of three equipment technologies ($E1$–$E3$) are assumed to be available for the processing sites. It is assumed that task $i1$ can be performed in equipment $E1$, task $i2$ in equipment $E2$, and task $i3$ in equipment $E3$. The discount rate has been taken equal to 35%. Input data associated to this example can be found in Appendix A.

The SC network configurations obtained by the traditional and flexible formulation are summarized in Figures 6 and 7. Numerical results show that the solution computed by the flexible formulation has higher performance in terms of NPV than the optimal traditional solution. Certainly, the optimal flexible model solution shows a NPV 22% greater than the one calculated by using the traditional approach. As shown in the figures, the traditional approach solution proposes to establish a processing site and a distribution center in one

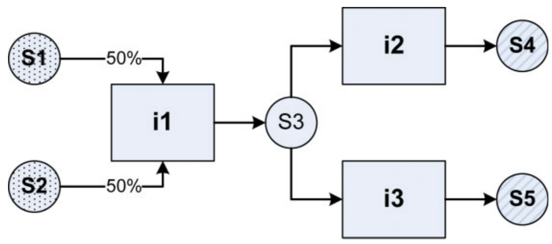
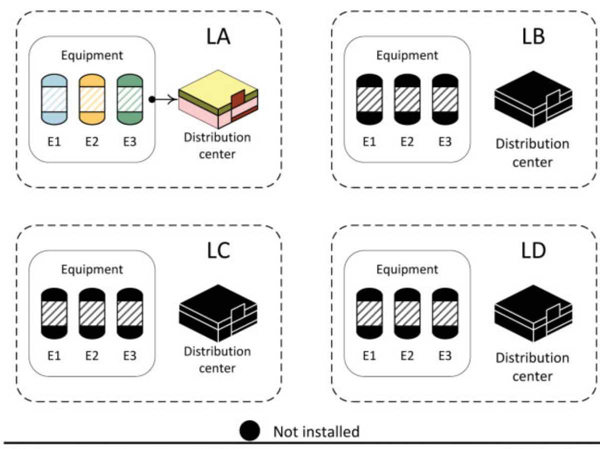



**Figure 5. STN notation for example 1.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

unique location (LA), while the flexible approach proposes to establish facilities in two locations (LA and LC). Let us notice that the flexible approach takes advantage of the capability of (i) splitting processes in different sites and (ii) transferring intermediate products among sites belonging to the same echelon type. Indeed, the flexible solution advises to install $E1$ at LA in order to produce state $S3$. A flow of state $S3$ exists from LA to LC. Equipment $E2$ and $E3$ are installed in LC where $S3$ is transformed into final products $S4$ and $S5$. Products are sent to the markets from LC. On the other hand, the traditional approach establishes all equipment technologies in the low-investment location LA because no intermediate material flows among facilities of same echelon type are allowed.

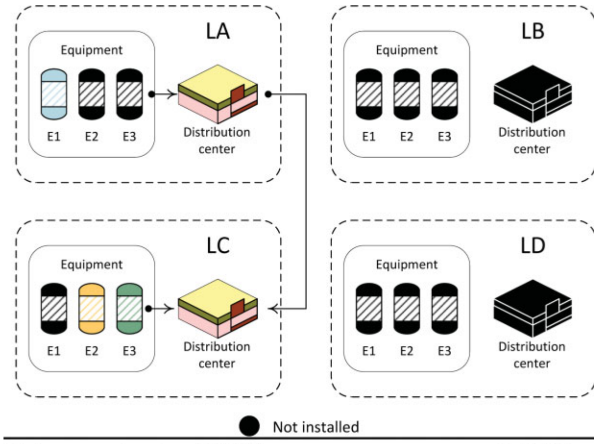
Figure 8 exhibits the economic characterization of each approach. As expected, the flexible model renders higher total transportation cost due to the transferring of $S3$;




| Net present value ($NPV$) | | | | | 479.086 m.u. |
|---|---|---|---|---|---|
| Capacity increment (c.u.) | | | | | |
| Manufacturing sites | | Equipment technology $j$ | | | |
| $l$ | $t$ | $E1$ | $E2$ | $E3$ | |
| $LA$ | 1 | 145.0 | 126.0 | 55.0 | |
| $LB$ | 1 | 0.0 | 0.0 | 0.0 | |
| $LC$ | 1 | 0.0 | 0.0 | 0.0 | |
| $LD$ | 1 | 0.0 | 0.0 | 0.0 | |
| Distribution centers | | | | | |
| | $t$ | $LA$ | $LB$ | $LC$ | $LD$ |
| | 1 | 50.0 | 0.0 | 0.0 | 0.0 |

**Figure 6. Traditional network design of illustrative example 1.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]




| Net present value ($NPV$) | | | | | 584.652 m.u. |
|---|---|---|---|---|---|
| Capacity increment (c.u.) | | | | | |
| Manufacturing sites | | Equipment technology $j$ | | | |
| $l$ | $t$ | $E1$ | $E2$ | $E3$ | |
| $LA$ | 1 | 163.0 | 0.0 | 0.0 | |
| $LB$ | 1 | 0.0 | 0.0 | 0.0 | |
| $LC$ | 1 | 0.0 | 142.0 | 62.0 | |
| $LD$ | 1 | 0.0 | 0.0 | 0.0 | |
| Distribution centers | | | | | |
| | $t$ | $LA$ | $LB$ | $LC$ | $LD$ |
| | 1 | 50.0 | 0.0 | 50.0 | 0.0 |

**Figure 7. Flexible network design of illustrative example 1.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

whereas the total production cost is lower because each equipment technology is installed to its corresponding lowest operating cost location. The lower production cost and higher revenue counterbalance the increase on investment and transportation costs.

This flexible model example consists of 4391 equations, 935 continuous variables, and 64 binary variables. The total CPU time is 0.34 CPU seconds and the integral optimal solution is found after 1434 iterations. The LP-relaxed solution gives a value of 1,762,204 m.u. for the objective function.

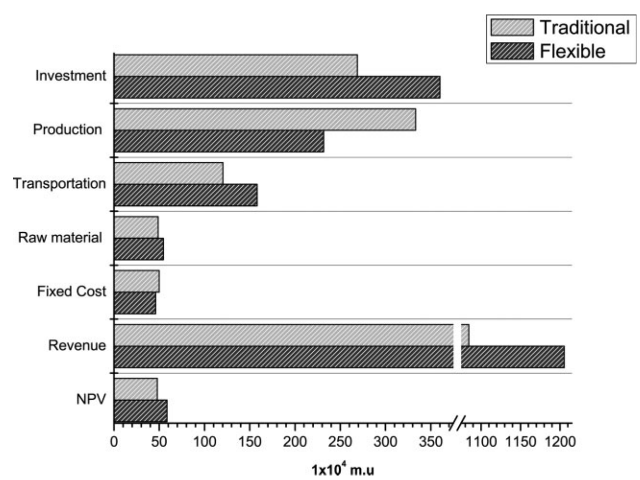



**Figure 8. Economic characterization of illustrative example 1.**

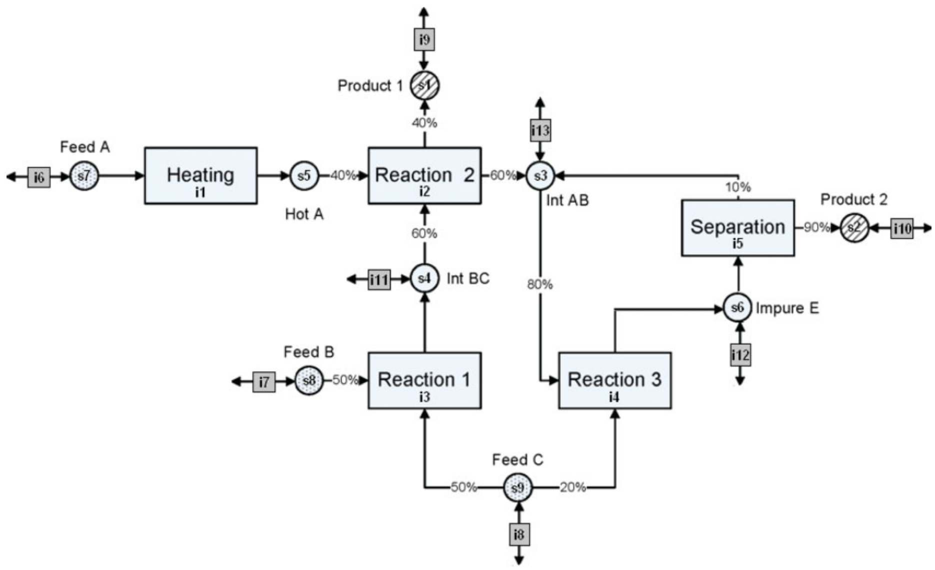
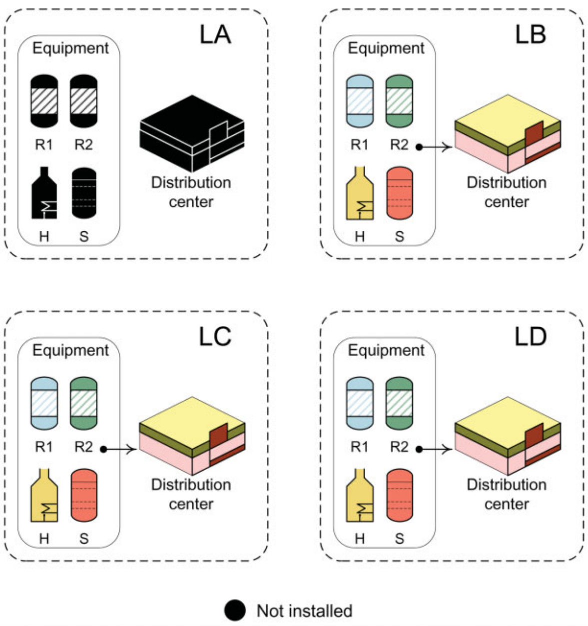



**Figure 9. STN notation for illustrative example 2.**

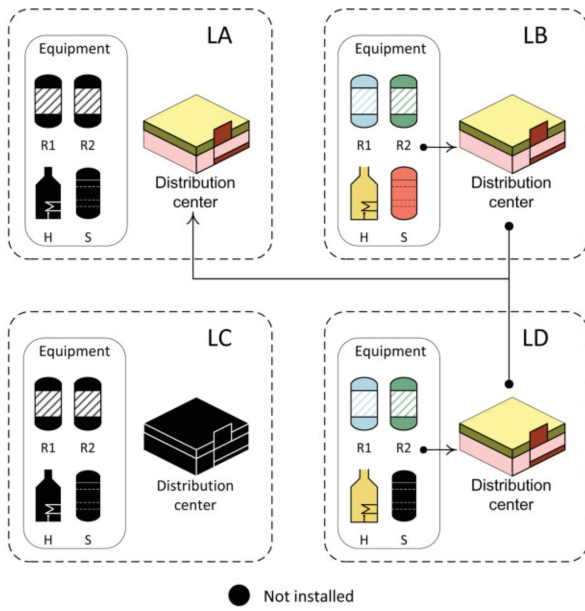
[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]




| **Net present value** ($NPV$) | | | | | 2.498.276 m.u. |
|---|---|---|---|---|---|
| Capacity increment (c.u.) | | | | | |
| Manufacturing sites | | Equipment technology $j$ | | | |
| $l$ | $t$ | $R1$ | $R2$ | $H$ | $S$ |
| $LA$ | 1 | 0.0 | 0.0 | 0.0 | 0.0 |
| $LB$ | 1 | 78.0 | 202.0 | 244.0 | 56.0 |
| $LC$ | 1 | 96.0 | 230.0 | 244.0 | 50.1 |
| $LD$ | 1 | 96.0 | 230.0 | 300.0 | 50.0 |
| Distribution centers | | | | | |
| | $t$ | $LA$ | $LB$ | $LC$ | $LD$ |
| | 1 | 0.0 | 51.0 | 62.0 | 62.0 |

**Figure 10. Traditional network design of illustrative example 2.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]




| **Net present value** ($NPV$) | | | | | 2.839.709 m.u. |
|---|---|---|---|---|---|
| Capacity increment (c.u.) | | | | | |
| Manufacturing sites | | Equipment technology $j$ | | | |
| $l$ | $t$ | $R1$ | $R2$ | $H$ | $S$ |
| $LA$ | 1 | 0.0 | 0.0 | 0.0 | 0.0 |
| $LB$ | 1 | 96.0 | 230.0 | 300.0 | 50.0 |
| $LC$ | 1 | 0.0 | 0.0 | 0.0 | 0.0 |
| $LD$ | 1 | 96.0 | 180.0 | 300.0 | 0.0 |
| Distribution centers | | | | | |
| | $t$ | $LA$ | $LB$ | $LC$ | $LD$ |
| | 1 | 50.0 | 62.0 | 64.0 | 0.0 |

**Figure 11. Flexible network design of illustrative example 2.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

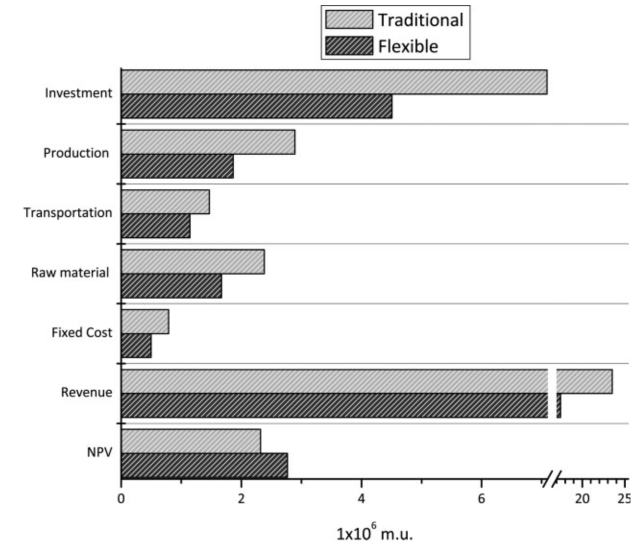


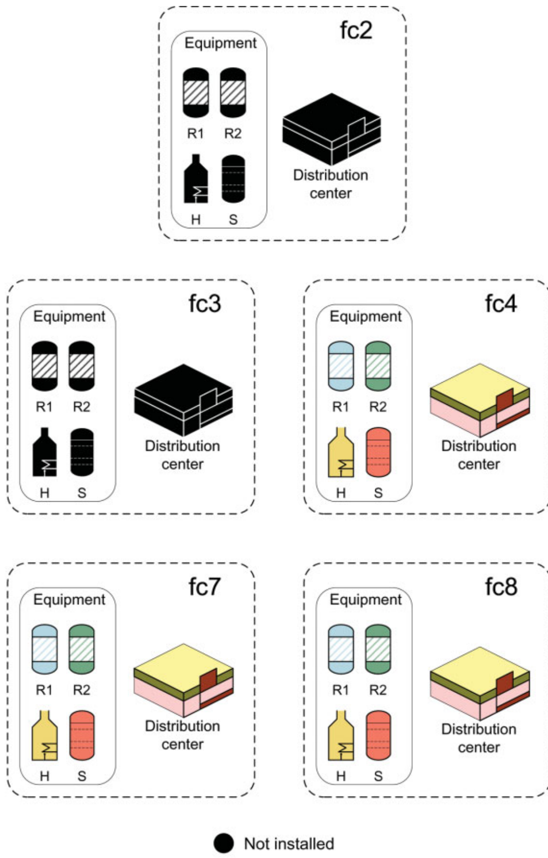

**Figure 12. Economic characterization of illustrative example 2.**

*Example 2.* A SC design-planning problem comprising four potential locations for the processing sites and the distribution centers in a planning horizon of five annual periods is considered. The STN representation of the process is depicted in Figure 9. Now, two final products ($S1$ and $S2$) and one intermediate product ($S3$) may be sold in six markets ($M1$–$M6$). $S7$, $S8$, and $S9$ are raw materials. A set of four equipment technologies ($H$, $R1$, $R2$, $S$) are available for the processing sites. It is assumed that task $i1$ can be performed in equipment $H$, task $i2$ in equipment $R1$, tasks $i3$ and $i4$ in equipment $R2$, and finally $i5$ in equipment $S$. The discount rate has been considered equal to 35%. Input data associated to this example can be found in Appendix B.

The resulting supply network configurations for this example are shown in Figures 10 and 11. In the traditional approach, plant sites and distribution centers are opened in three different locations (LB, LC, and LD). For this approach, the four equipment technologies are installed at every location. On the other hand, the flexible model establishes two plant sites at locations LB and LD. In location LB, all equipment technologies are installed while the separation technology $S$ is not installed at location LD. The flexible model opens three distributions centers which are located at LA, LB, and LC. In the flexible solution, a flow of material $S3$ appears. $S3$ is transferred from distributions centers located at LB and LD to the distribution center located at LA.

The flexible approach solution results in an improvement of 13.7% over the traditional approach. Figure 12 shows the economic results of both solutions. As one can notice, the flexible solution leads to cost reductions, but it also results in a revenue reduction of $\sim 6 \times 10^6$ m.u. Consequently, the profit contribution to the NPV is reduced about 25% by using the flexible approach. However, the flexible model investment on facilities is 36.5% lower in comparison to the traditional one. Therefore, the SC configuration proposed by the flexible model renders a better economic performance. The flexible model for this example consists of 3117 equations, 26,800 continuous variables and 136 binary variables. The total CPU time is 12.41 CPU seconds and the integral optimal solution is found after 20,310 iterations. The LP-relaxed solution gives a value of 6,587,803 m.u. for the objective function.

*Example 3.* Here, a more complex SC design-planning problem of eight potential locations ($fc1$–$fc8$) for the processing sites and the distribution centers is addressed. A planning horizon of 48 month periods is examined. The production process is the same of example 2 which is depicted in Figure 9. Two final products ($S1$ and $S2$) and two intermediate product ($S3$ and $S4$) may be sold in six markets ($fc12$–$fc17$). $S7$, $S8$, and $S9$ are raw materials. A set of four

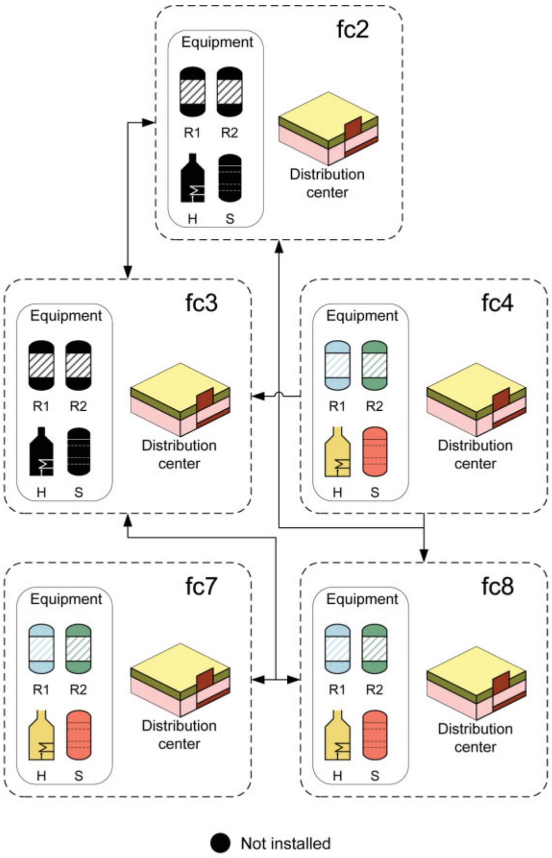



| **Net present value** ($NPV$) | | | | | 70.883.086 m.u. |
|---|---|---|---|---|---|
| | | Capacity increment (c.u.) | | | |
| Manufacturing sites | | Equipment technology $j$ | | | |
| $fc$ | $t$ | $R1$ | $R2$ | $H$ | $S$ |
| $fc4$ | 1 | 66.37 | 243.41 | 207.40 | 83.31 |
| $fc7$ | 1 | 73.15 | 250.00 | 228.60 | 86.33 |
| $fc8$ | 1 | 80.00 | 250.00 | 250.00 | 69.36 |
| | 13 | 50.48 | 146.41 | 157.75 | 55.85 |

| Distribution centers | | | | | |
|---|---|---|---|---|---|
| $t$ | $fc2$ | $fc3$ | $fc4$ | $fc7$ | $fc8$ |
| 1 | 0.00 | 0.00 | 49.69 | 55.48 | 62.57 |
| 13 | 0.00 | 0.00 | 0.00 | 0.00 | 29.81 |

**Figure 13. Traditional SC network design of illustrative example 3.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]




| Net present value ($NPV$) | | | | | 73.383.512 m.u. |
|---|---|---|---|---|---|
| | | Capacity increment (c.u.) | | | |
| Manufacturing sites | | Equipment technology $j$ | | | |
| $fc$ | $t$ | $R1$ | $R2$ | $H$ | $S$ |
| $fc4$ | 1 | 74.26 | 250.00 | 232.05 | 59.49 |
| | 13 | 0.00 | 0.00 | 0.00 | 25.00 |
| $fc7$ | 1 | 70.42 | 250.00 | 220.06 | 79.89 |
| | 13 | 0.00 | 0.00 | 0.00 | 25.00 |
| $fc8$ | 1 | 80.00 | 250.00 | 250.00 | 97.62 |
| | 13 | 45.32 | 139.47 | 141.63 | 32.40 |
| Distribution centers | | | | | |
| $t$ | $fc2$ | $fc3$ | $fc4$ | $fc7$ | $fc8$ |
| 1 | 25,00 | 25,00 | 55,05 | 52,35 | 75,83 |
| 13 | 0,00 | 0,00 | 0,00 | 0,00 | 25,00 |

**Figure 14. Flexible SC network design of illustrative example 3.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

equipment technologies ($H$, $R1$, $R2$, $S$) are available for the processing sites. It is assumed that task $i1$ can be performed in equipment $H$, task $i2$ in equipment $R1$, tasks $i3$ and $i4$ in equipment $R2$, and finally $i5$ in equipment $S$. The discount rate has been taken equal to 35%. Example 3 input data can be found in Appendix C.

The distinct configurations of the traditional SC design-planning and the flexible model are shown in Figures 13 and 14. In the traditional SC design-planning case, three facility locations are established; $fc4$, $fc7$, and $fc8$. Processing plants and distribution centers are installed at all established sites. In the flexible SC design-planning case, five facility locations are established; $fc2$, $fc3$, $fc4$, $fc7$, and $fc8$. Processing plants and distribution centers are installed at sites: $fc4$, $fc7$, and $fc8$ while in sites $fc2$ and $fc3$ are established only distribution centers. The attached tables of Figures 13 and 14 illustrate the capacity increase for every equipment technology installed in each planning period at each manufacturing site. The main flexible design-planning model characteristic is the significant material flow between production plants. Moreover, material flows between distribution centers also exist. Figure 14 clearly depicts the aforementioned flow connections between the SC echelons.

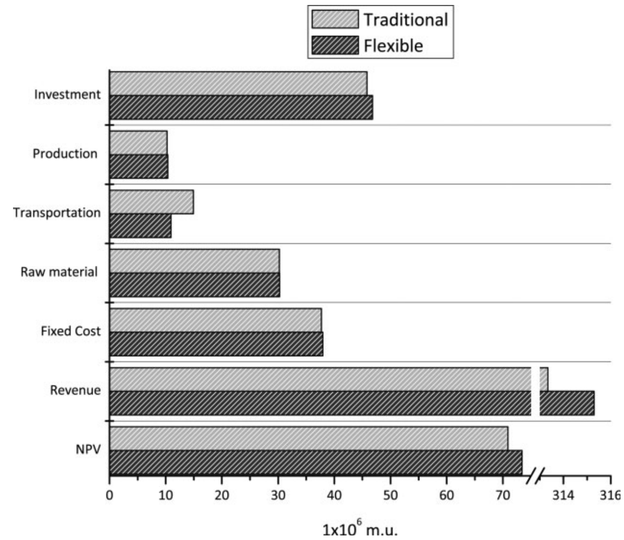



**Figure 15. Economic characterization of example 3.**

In Figure 15 follows the economic characterization of both cases. The flexible approach results into investment and fixed costs slightly higher than the traditional approach. However, the flexible SC design-planning gives significantly lower transportation costs. This constitutes the main fact that contributes to higher revenue and NPV values. Figure 16 presents the NPV contribution per semester of both cases. The flexible SC design-planning model gives better NPV for all semesters except for the first one where the capital investment is higher (two additional sites are established) than the one of the traditional approach. Finally, it is worth mentioning that the traditional approach results into a NPV

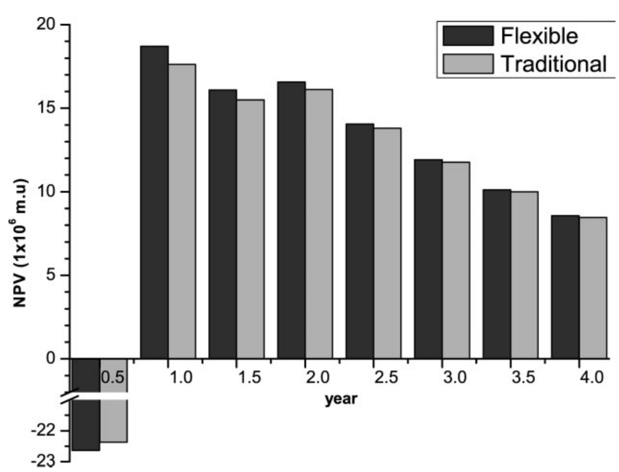



**Figure 16. NPV contribution per semester for example 3.**

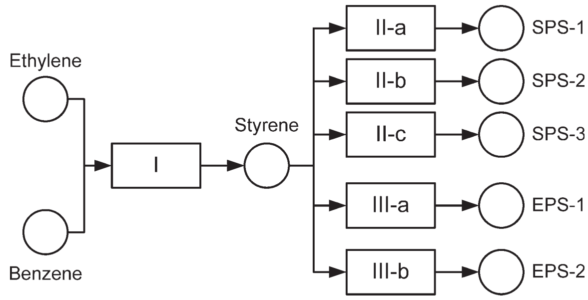



**Figure 17. STN for the case study.**

equal to 70,883,086 m.u. while the flexible one gives a NPV equal to 73,383,512 m.u.; a 3.53% of improvement. This larger flexible model example consists of 29,825 equations, 223,185 continuous variables, and 992 binary variables. The total CPU time is 1980 CPU seconds and the integral optimal solution is found after 495,714 iterations. The LP-relaxed solution gives a value of 91,011,406.61 m.u. for the objective function.

### *Case study*

Consider the following case study which is an adapted version of the one introduced by You and Grossmann.[8] This case study was motivated by a real world application concerning a polystyrene SC design. The polystyrene production process is shown in Figure 17. Styrene monomers are produced from ethylene and benzene, then styrene is processed to obtain five final products: three different types of solid polystyrene (SPS) and two types of expandable polystyrene (EPS). Potential benzene suppliers are located in Texas (TX), Louisiana (LA), and Alabama (AL); while ethylene suppliers are located in Illinois (IL), TX, and Mississippi (MS). Customers are aggregated into nine sale regions according to their geographical proximity. Distribution centers and processing plants may be established in eight different states which are Michigan (MI), TX, California (CA), LA, Nevada (NV), Georgia (GA), Pennsylvania (PA), and Iowa (IA). Figure 18 shows the SC components location. The potential SC superstructure that was considered by You and Grossmann is depicted in Figure 19.

The case study has been modified in order to consider all production activities in every potential facility location. In order to exclude biased data, cost associated to those activities that are not considered in the given pre-established superstructure have been randomly determined by using a uniform probability distribution whose lower limit is equal to the superstructure most costly option. Following this procedure, those options which have not been included in the superstructure will imply a higher cost/investment. Besides, transportation cost has been calculated taking into consideration the distances between facilities, sales regions, and suppliers. The problem has been solved for a planning horizon of 48 monthly periods. Discount rate for this case study has been taken equal to 20%.

Figure 20 shows the optimal SC configuration obtained using the pre-established superstructure. The traditional approach proposes to produce styrene in the sites located at TX and LA, while equipment technologies to produce final products are installed at MI, CA, and LA. Two intersite material flows are included from TX to CA and from LA to MI for the styrene monomer shipment. This SC network configuration supplies the customers from four different distribution centers. In the flexible approach case, the optimal SC configuration involves three production sites (TX, CA, and LA) and three distribution centers (TX, GA, and IA) as shown in Figures 21 and 22. An intersite styrene flow exists from TX site to LA site. It is interesting to observe that the flexible solution has established a distribution center at TX that transfers final products to customers located in Washington (WA) but it also receives raw materials from

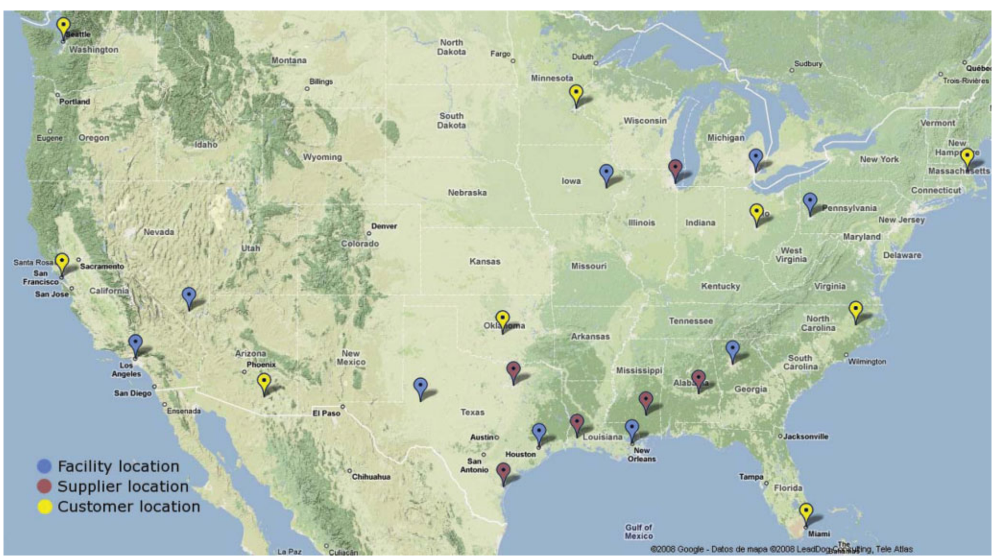



**Figure 18. Location map for the case study.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

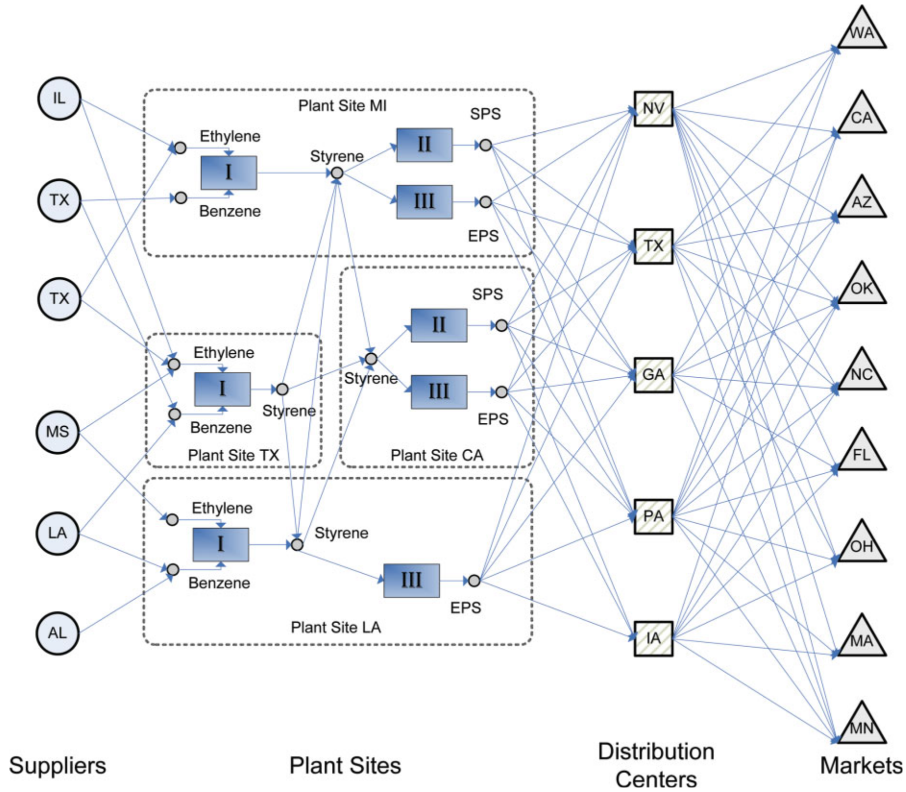



**Figure 19. Potential process SC network superstructure for the case study given in a traditional approach.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

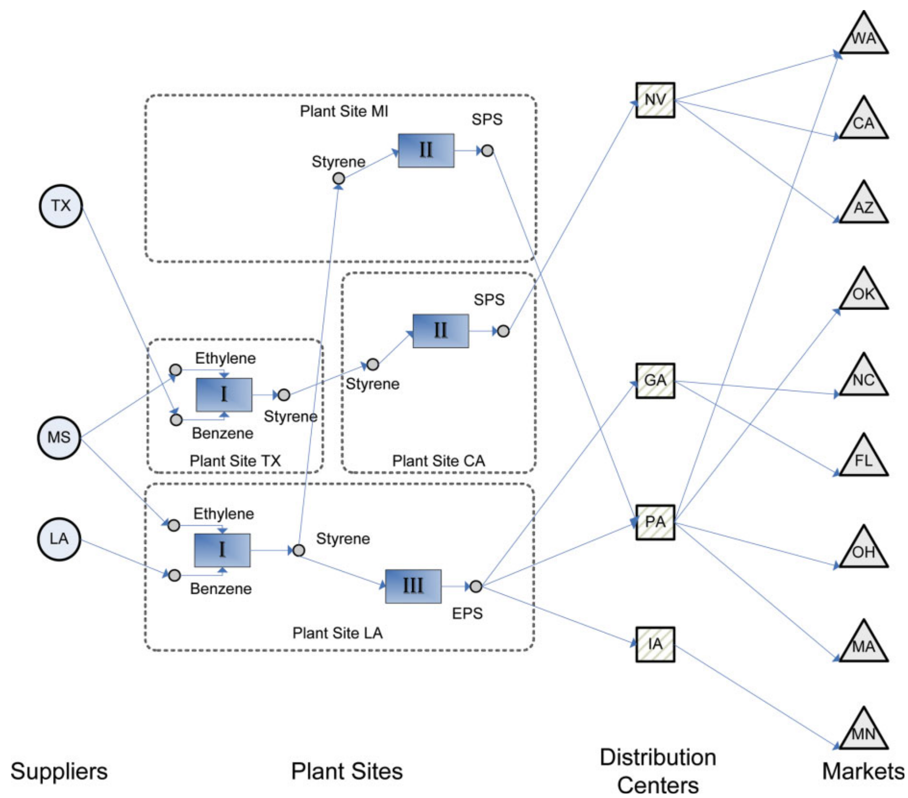



**Figure 20. Optimal SC network design for the case study considering the pre-established superstructure, NPV = 676 × $10^6$ m.u.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

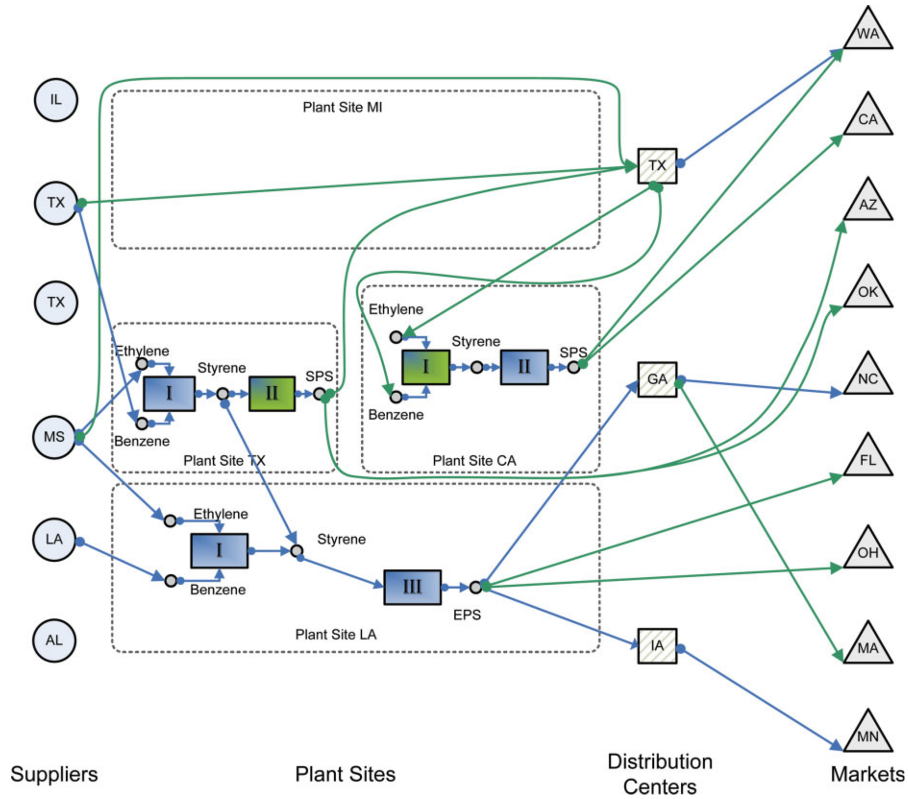



**Figure 21. Optimal SC network design for the case study by using the flexible approach, NPV = 1180 × 10$^6$ m.u.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

suppliers to be sent to the production site at CA. Raw materials are usually not considered to be handled by distribution centers, however improvements can be achieved by doing so. Moreover, production plants are directly shipping final products to some markets which has not been considered in the SC superstructure. Additionally, it should be noticed that technologies I and II are installed at CA and TX sites, respectively. However, the allocation of these technologies to those sites is not considered in the SC superstructure (see Figure 19). The flexible approach has installed them in spite of being more costly than the other available options, because the trade-off between the required higher investment


**Figure 22. SC network in the location map for the flexible solution.**

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

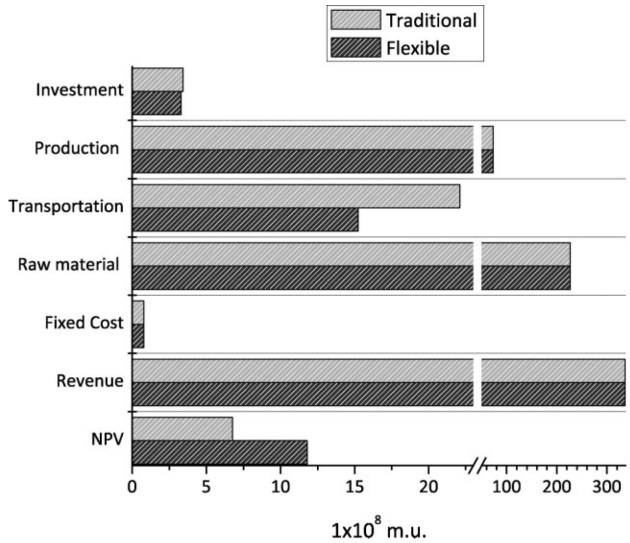



**Figure 23. Case study economic characterization.**

and the transportation cost results in an overall NPV increase. The economic outcomes of both solutions are presented in Figure 23. The flexible approach solution leads to an NPV improvement of 74.6% over the traditional approach. Such improvement is achieved mostly by reducing the total transportation cost. This case study flexible model consists of 43,027 equations, 693,025 continuous variables, and 1008 binary variables. The total CPU time is 14 CPU seconds and the integral solution is obtained after 24,412 iterations. The LP-relaxed solution gives a value of 1186 $\times$ $10^6$ m.u. for the objective function.

## Final Considerations

This work has addressed the strategic problem of designing a SC network. The proposed approach utilizes a SC design and planning model that permits material flows of any kind (raw and intermediate materials, final products) between any kind of facilities. What is more, only potential locations are provided to the model as input data; decisions regarding the installation of a processing plant, a distribution center or both of them at a location are made during the optimization procedure. It is noteworthy that a main feature of the flexible model is that it does not require a pre-established process network superstructure thus allowing to optimally define the sub-trains in which production process is decoupled and their respective locations. As a result, processing facilities outputs may be intermediate materials. This is one key feature in modeling the complex global SCs behavior.

We consider that the examples presented have evidenced that a great potential to improve the firm's economic performance can be gained by exploring the whole range of available alternatives when designing a SC. The flexible model proposed in this work enables to do this exploration in a straightforward manner. In the worst scenario, the flexible approach would find the same optimal solution as traditional ones. The flexible model can be also implemented to exploit flexibility in strict operations planning when the SC network configuration is already fixed. For this case, the model will support task allocation and the definition of links among network nodes taking into account capacity constraints.

Shah[19] states that it has not really been shown what is an appropriate description of manufacturing processes at the SC level. By translating the STN concept to the whole SC environment an adequate representation of all operations and materials entailed in a production system can be achieved. It should be also emphasized that the proposed model simplifies the representation of batch and/or continuous process SCs into the same framework. Moreover, given that the proposed SC model is an extension of a classical multipurpose plant scheduling formulation, it may eventually facilitate the consideration of scheduling decisions while designing a SC. This issue constitutes one important subject of our current research.

While dealing with SC design problems, managers usually have considerable more time availability to come out with a final decision (i.e., SC configuration) than in the case of planning and scheduling problems. Despite this fact it is necessary to devote research efforts to develop decomposition strategies in order to tackle industrial scale problems. In this regard, spatial and temporal decompositions schemes based on Lagrangean decomposition have proven to considerably reduce the computational burden associated with the solution of this kind of problems.[25] Future work is focused in developing decomposition schemes applicable to the model presented in this article.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## Notation

### *Indices*

- $e$ = suppliers
- $f$ = facility locations
- $i$ = tasks
- $j$ = plant equipment
- $s$ = states
- $t$ = planning periods

### *Sets*

- $E_s$ = set of suppliers $e$ that provide raw material $s$
- $\hat{E}_{\text{prod}}$ = set of suppliers $e$ that provide production services
- $\tilde{E}_{\text{tr}}$ = set of suppliers $e$ that provide transportation services
- FP = set of states $s$ that are final products
- $\hat{J}_f$ = equipment $j$ that can be installed at location $f$
- $J_i$ = equipment that can perform task $i$
- $M$ = set of market locations
- $R_f$ = set of raw materials that can be provided from location $f$
- RM = set of states $s$ that are raw materials
- $\text{RM}_e$ = set of raw materials that are offered by supplier $e$
- $S^I$ = set of unstable materials which cannot be transfered between facilities
- Sup = set of supplier locations
- $T_r$ = set of distribution tasks
- $T_s$ = set of tasks producing state $s$
- $\overline{T}_s$ = set of tasks consuming state $s$

### Parameters

$A_{sft}$ = maximum availability of raw material $s$ in period $t$ at location $f$
$\text{Dem}_{sft}$ = product $s$ demand at market $f$ in period $t$
$\text{FCFJ}_{jft}$ = fixed cost per unit of capacity of plant equipment $j$ at location $f$ in period $t$
$\text{FCFS}_{ft}$ = fixed cost per unit of distribution center capacity at location $f$ in period $t$
$I_{ft}^{\text{J}}$ = investment required to establish a processing facility in location $f$ in period $t$
$I_{ft}^{\text{S}}$ = investment required to establish a distribution center in location $f$ in period $t$
$\text{MinCSL}_s$ = lower bound of customer service level for product $s$
$\text{Price}_{sft}$ = price of product $s$ at market $f$ in period $t$
$\text{Price}_{jft}^{FJ}$ = investment required per unit of capacity of equipment $j$ increased at facility $f$ in period $t$
$\text{Price}_{ft}^{FS}$ = investment required per unit of distribution center capacity increased at facility $f$ in period $t$
rate = discount rate

### Binary variables

$\text{JB}_{ft}^{l}$ = 1 if a processing site at location $f$ is established in period $t$, 0 otherwise
$\text{SB}_{ft}^{l}$ = 1 if a distribution center at location $f$ is established in period, 0 otherwise $t$
$V_{jft}$ = 1 if the equipment $j$ capacity is increased at location $f$ in period $t$ 0, otherwise
$X_{ft}$ = 1 if the distribution center capacity at location $f$ is increased at period $t$, 0 otherwise

### Continuous variables

$\text{EPurch}_{et}$ = economic value of purchases executed in period $t$ to supplier $e$
$\text{ESales}_t$ = economic value of sales executed in period $t$
$\text{FAsset}_t$ = investment on fixed assets in period $t$
$\text{FCost}_t$ = fixed cost in period $t$
$\text{FS}_{ft}$ = total distribution center capacity at location $f$ during period $t$
$\text{FSE}_{ft}$ = distribution center capacity increment at location $f$ during period $t$
$\text{FJ}_{jft}$ = plant equipment $j$ total capacity during period $t$ at location $f$
$\text{FJE}_{jft}$ = plant equipment $j$ capacity increment at location $f$ during period $t$
NPV = net present value
$P_{ijff't}$ = production rate of task $i$ in equipment $j$ in period $t$ whose origin is location $f$ and destination location $f'$
$\text{Profit}_t$ = profit achieved in period $t$
$\text{Purch}_{et}^{\text{pr}}$ = amount of money payable to supplier $e$ in period $t$ associated with production activities
$\text{Purch}_{et}^{\text{rm}}$ = amount of money payable to supplier $e$ in period $t$ associated with raw materials consumption
$\text{Purch}_{et}^{\text{tr}}$ = amount of money payable to supplier $e$ in period $t$ associated with transport services
$\text{Purch}_{esft}$ = amount of raw material $s$ purchased to supplier $e$ from location $f$ in period $t$
$\text{Sales}_{sff't}$ = amount of product $s$ sold from location $f$ in market $f'$ in period $t$
$S_{sft}$ = amount of state $s$ stock at location $f$ in period $t$

### Greek letters

$\alpha_{sij}$ = mass fraction of task $i$ for state $s$ production in equipment $j$
$\overline{\alpha}_{sij}$ = mass fraction of task $i$ for state $s$ consumption in equipment $j$
$\beta_{jf}$ = minimum utilization rate of plant equipment $j$ capacity that is allowed at location $f$
$\theta_{ijff'}$ = capacity utilization factor of task $i$ performed in plant equipment $j$ whose origin is location $f$ and destination location $f'$
$\pi_{jf}$ = time to install and set up equipment $j$ in facility $f$
$\hat{\pi}_f$ = time to install and set up a distribution center in location $f$
$\rho_{eff'}^{\text{tr}}$ = unitary transportation costs from location $f$ to location $f'$
$\tau_{ijfe}^{\text{ut1}}$ = unitary cost associated with task $i$ performed in equipment $j$ from location $f$ and payable to external supplier $e$
$\tau_{sfe}^{\text{ut2}}$ = unitary cost associated with handling the inventory of material $s$ in location $f$ and payable to external supplier $e$
$\upsilon_s$ = specific volume of product $s$
$\psi_{est}$ = raw material unitary cost $s$ offered by external supplier $e$ in period $t$

### Superscripts

L = lower bound
U = upper bound

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## Appendix A: Illustrative Example 1 Input Data

**Table A1. Establishing Inversion Costs (m.u.)**

| Site | Plant | Distribution Center |
|---|---|---|
| LA | 62,250 | 133579.6 |
| LB | 1,182,750 | 270787.5 |
| LC | 996,000 | 115733.3 |
| LD | 933,750 | 369765.0 |

**Table A2. Sale Prices (euros)**

| | Markets | |
|---|---|---|
| Product | *M*1 | *M*2 |
| s4 | 540 | 492 |
| s5 | 720 | 762 |

**Table A3. Products Demands (tns)**

| | Periods | | | | |
|---|---|---|---|---|---|
| Market | *t*1 | *t*2 | *t*3 | *t*4 | *t*5 |
| Product $S = S4$ | | | | | |
| *M*1 | 2108 | 1784 | 3290 | 3737 | 3077 |
| *M*2 | 727 | 733 | 765 | 1288 | 558 |
| Product $S = S5$ | | | | | |
| *M*1 | 740 | 791 | 1030 | 826 | 829 |
| *M*2 | 358 | 595 | 1041 | 831 | 624 |

**Table A4. Raw Materials Data**

| Raw Material | Purchase Price (m.u.) | Supplier Capacity (tns) |
|---|---|---|
| *s*1 | 30 | 2500 |
| *s*2 | 24 | 2000 |

**Table A5. Equipment Technology Data**

| | Equipment | $FJE^L$ (tns) | $FJE^U$ (tns) | Installation Cost (m.u.) | Fixed Cost (m.u.) |
|---|---|---|---|---|---|
| E1 | Reactor I | 25 | 250 | 4200 | 168 |
| E2 | Reactor II | 25 | 250 | 8000 | 320 |
| E3 | Reactor III | 25 | 250 | 16,000 | 640 |

**Table A6. Transportation Costs (m.u./tn)**

| | Sitess | | | | Markets | |
|---|---|---|---|---|---|---|
| Site | LA | LB | LC | LD | *M*1 | *M*2 |
| LA | 0 | 157.6 | 36 | 191.2 | 68 | 62.4 |
| LB | 157.6 | 0 | 51.2 | 59.2 | 140 | 124 |
| LC | 36 | 51.2 | 0 | 71.2 | 33.6 | 84 |
| LD | 191.2 | 59.2 | 71.2 | 0 | 96 | 137.6 |

**Table A7. Capacity Utilization Rates ($\theta_{ijff'}$)**

| Task | Equipment | $\theta_{ijff}{}'$ |
|---|---|---|
| *i*1 | E1 | 0.04 |
| *i*2 | E2 | 0.05 |
| *i*3 | E3 | 0.05 |

## Appendix B: Illustrative Example 2 Input Data

**Table B1. Establishing Inversion Costs (m.u.)**

| Site | Plant | Distribution Center |
|---|---|---|
| LA | 2,387,000 | 127,050 |
| LB | 1,155,707 | 432,400 |
| LC | 1,222,555 | 929,605 |
| LD | 1,155,000 | 693,000 |

**Table B2. Sale Prices (m.u.)**

| | Markets | | | | | |
|---|---|---|---|---|---|---|
| Product | *M*1 | *M*2 | *M*3 | *M*4 | *M*5 | *M*6 |
| *s*1 | 275.00 | 286.00 | 247.50 | 302.50 | 264.00 | 363.00 |
| *s*2 | 240.00 | 248.00 | 220.00 | 244.00 | 268.00 | 211.00 |
| *s*3 | 231.00 | 220.00 | 302.50 | 192.50 | 247.50 | 302.50 |

**Table B3. Products Demands (tns)**

| | Periods | | | | |
|---|---|---|---|---|---|
| Market | *t*1 | *t*2 | *t*3 | *t*4 | *t*5 |
| Product $S = S1$ | | | | | |
| *M*1 | 6763 | 8478 | 6895 | 8014 | 11294 |
| *M*2 | 4217 | 3569 | 6581 | 7475 | 6155 |
| *M*3 | 2907 | 2931 | 3057 | 5148 | 2233 |
| *M*4 | 2275 | 3632 | 3661 | 6331 | 5182 |
| *M*5 | 1829 | 1805 | 1400 | 2393 | 2817 |
| *M*6 | 1806 | 2414 | 3572 | 4378 | 3216 |
| Product $S = S2$ | | | | | |
| *M*1 | 1666 | 3099 | 2713 | 3340 | 1608 |
| *M*2 | 352 | 465 | 589 | 373 | 608 |
| *M*3 | 2210 | 1928 | 1105 | 1794 | 1240 |
| *M*4 | 1843 | 2557 | 2305 | 1925 | 2216 |
| *M*5 | 812 | 1396 | 840 | 1009 | 1195 |
| *M*6 | 594 | 931 | 630 | 1184 | 1092 |
| Product $S = S3$ | | | | | |
| *M*1 | 2542 | 3962 | 6054 | 5933 | 4477 |
| *M*2 | 1149 | 1017 | 1003 | 1808 | 1936 |
| *M*3 | 1783 | 1162 | 1946 | 2204 | 2868 |
| *M*4 | 1333 | 1384 | 2181 | 1828 | 845 |
| *M*5 | 1006 | 630 | 686 | 971 | 1176 |
| *M*6 | 459 | 542 | 511 | 240 | 680 |

**Table B4. Raw Materials Data**

| Raw Material | Purchase Price (m.u.) | Supplier Capacity (tns) |
|---|---|---|
| *s*7 | 25 | 2500 |
| *s*8 | 20 | 2000 |
| *s*9 | 37.5 | 2250 |

**Table B5. Equipment Technology Data (m.u./c.u.)**

| | Equipment | $FJE^L$ | $FJE^U$ | Unitary Installation Cost |
|---|---|---|---|---|
| *j*1 | Heater (H) | 50 | 300 | 220.70 |
| *j*2 | Reactor (R1) | 50 | 300 | 662.20 |
| *j*3 | Reactor (R2) | 50 | 300 | 882.90 |
| *j*4 | Separator (S) | 50 | 300 | 1655.50 |

Table B6. Transportation Costs (m.u./tns)

| | Sites | | | | Markets | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Site | LA | LB | LC | LD | *M*1 | *M*2 | *M*3 | *M*4 | *M*5 | *M*6 |
| LA | 0 | 33.6 | 26.4 | 33 | 54 | 84 | 1.2 | 84.6 | 75 | 71.4 |
| LB | 33.6 | 0 | 18 | 48.6 | 104.4 | 106.8 | 141.6 | 70.2 | 1.2 | 35.4 |
| LC | 26.4 | 18 | 0 | 30.6 | 65.4 | 69.6 | 36.6 | 106.2 | 126 | 1.2 |
| LD | 33 | 48.6 | 30.6 | 0 | 1.2 | 64.8 | 69.6 | 124.2 | 1.2 | 47.4 |

## Appendix C: Illustrative Example 3 Input Data

Table C1. Establishing Inversion Costs (m.u.)

| Site | Plant | Distribution Center |
|---|---|---|
| *fc*1 | 18,060,000 | 2600 |
| *fc*2 | 12,180,000 | 870,000 |
| *fc*3 | 13,020,000 | 3850 |
| *fc*4 | 9,240,000 | 924,000 |
| *fc*5 | 5,460,000 | 11,50,000 |
| *fc*6 | 5,880,000 | 1,050,000 |
| *fc*7 | 5,460,000 | 2,250,000 |
| *fc*8 | 6,300,000 | 1,000,000 |

Table C2. Sale Prices (m.u.)

| | Markets | | | | | |
|---|---|---|---|---|---|---|
| Product | *fc*12 | *fc*13 | *fc*14 | *fc*15 | *fc*16 | *fc*17 |
| *s*1 | 600 | 624 | 540 | 660 | 576 | 720 |
| *s*2 | 240 | 248 | 220 | 244 | 268 | 192 |
| *s*3 | 168 | 160 | 220 | 140 | 180 | 200 |
| *s*4 | 144 | 168 | 120 | 160 | 140 | 150 |

Table C3. Products Demands (tns)

| | Markets | | | | | |
|---|---|---|---|---|---|---|
| Product | *fc*12 | *fc*13 | *fc*14 | *fc*15 | *fc*16 | *fc*17 |
| *s*1 | 5701 | 3665 | 3398 | 2686 | 1342 | 2310 |
| *s*2 | 2116 | 365 | 1468 | 1355 | 910 | 630 |
| *s*3 | 3246 | 887 | 1369 | 1353 | 609 | 410 |
| *s*4 | 2582 | 295 | 3016 | 466 | 556 | 700 |

Table C4. Raw Materials Data

| Raw Material | Purchase Price (m.u.) | Supplier Capacity (tns) |
|---|---|---|
| *s*7 | 30 | 2500 |
| *s*8 | 24 | 2000 |
| *s*9 | 45 | 2250 |

Table C5. Equipment Technology Data

| | Equipment | $FJE^L$ | $FJE^U$ (tns) | Installation Cost (m.u.) | Fixed Cost (euros) |
|---|---|---|---|---|---|
| *j*1 | Heater | 25 | 250 | 3010 | 120.4 |
| *j*2 | Reactor | 25 | 250 | 9030 | 361.2 |
| *j*3 | Reactor | 25 | 250 | 12,040 | 481.6 |
| *j*4 | Separator | 25 | 250 | 4515 | 180.6 |

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Table C6. Transportation Costs (m.u./tn)

| | Sites | | | | | | | | Markets | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Site | *fc*1 | *fc*2 | *fc*3 | *fc*4 | *fc*5 | *fc*6 | *fc*7 | *fc*8 | *fc*12 | *fc*13 | *fc*14 | *fc*15 | *fc*16 | *fc*17 |
| *fc*1 | 0 | 263.98 | 201 | 320.26 | 97.82 | 140.7 | 180.9 | 249.24 | 0 | 83.08 | 108.54 | 71.02 | 68.34 | 75.04 |
| *fc*2 | 263.98 | 0 | 85.76 | 99.16 | 178.22 | 159.46 | 144.72 | 127.3 | 87.1 | 0 | 170.18 | 103.18 | 97.82 | 103.18 |
| *fc*3 | 201 | 85.76 | 0 | 119.26 | 105.86 | 75.04 | 58.86 | 73.7 | 120.6 | 187.6 | 0 | 188.94 | 167.5 | 159.46 |
| *fc*4 | 320.26 | 99.16 | 119.26 | 0 | 223.78 | 186.26 | 152.76 | 92.46 | 151.42 | 203.68 | 243.88 | 0 | 191.62 | 192.96 |
| *fc*5 | 97.82 | 178.22 | 105.86 | 223.78 | 0 | 46.9 | 87.1 | 152.76 | 253.26 | 289.44 | 349.74 | 222.44 | 0 | 76.38 |
| *fc*6 | 140.7 | 159.46 | 75.04 | 186.26 | 46.9 | 0 | 40.2 | 108.54 | 233.16 | 238.52 | 316.24 | 156.78 | 0 | 79.06 |
| *fc*7 | 180.9 | 144.72 | 58.96 | 152.76 | 87.1 | 40.2 | 0 | 68.34 | 146.06 | 155.44 | 81.74 | 237.18 | 281.4 | 0 |
| *fc*8 | 249.24 | 127.3 | 73.7 | 92.46 | 152.76 | 108.54 | 68.34 | 0 | 0 | 144.72 | 155.44 | 277.38 | 0 | 105.86 |